International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 2, Mar 2014, 37-38


International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

# NOTE ON GLOBAL SOLVABILITY

**T. VENKATESH[1], MITA N AMONKAR[2] & M. E. SHIVKUMAR[3]**

[1]Department of Mathematics, Rani Channama University, Belgaum, Karnataka, India

[2,3]Research Scholars, Department of Mathematics, Rani Channama University, Belgaum, Karnataka, India

## ABSTRACT

The connection between PDEs and Geometry is well known. When the flow on smooth manifold is characterized as vector field then one would expect that the flow remains stable for time as much possible as we wish. But, this is too much a requirement several topological constraints will come in the way of such smooth flows. We have made a note of this fact interms of global solvability.

**KEYWORDS:** Smooth Manifolds, Flows and Finite Time Singularities

## 1. INTRODUCTION

In this paper we consider the problem of global solvability from the point of view of geometry. The problem is basically to have a control over the shape of a surface or more generally a hyper surface(manifolds). In the theory of surfaces this amounts to believe that the second fundamental form and the corresponding curvatures associated with the Gauss map are efficient tools for controlling the shape of a surface. Then, one also has the theory of integration developed on compact surfaces with the usual properties of Lebegue integration. These are the main bases for carrying out a deeper study of surfaces in $\mathbb{R}^n$, n $\geq 3$. From the point of view of modern differential geometry, influenced most of the time by topology, it is the global view point; Brouwer- Samuelson theorem, Jordon- Brouwer separation theorem and Brouwer fixed point theorem are driven by the topological notions. But, in the perspective of Differential geometry and whenever one thinks about global problems in geometry, usually one would impose some global condition to avoid the case that when pieces of larger surfaces may appear as possible solutions, that is surfaces that are proper open subsets of another larger surface. Compactness is one such a topological requirements. A weaker notion is the geodesic completeness. Confining closed subsets of $\mathbb{R}^3$, that surface as subset of Euclidean spaces and compact, the gain would be some analytical methods either available or can be developed on them.

## 2. PROBLEM

The first problem that we have considered is the question that are already posed in case of curves, is to determine compact surfaces compact surfaces with their curvatures having some specific behavior.

For instance, one can characterize compact surfaces with constant Guass or mean curvature? Guass curvature was studied by F. Minding in 1838 with a purpose to relate them with non- Euclidean geometries. For compact case the only example is the sphere (Hilbert- Liebmann theorem) first proved by Liebmann in 1899 and D. Liebmann in 1901. A natural generalization to this question is to consider compact surfaces whose Gaussian curvature does not change its sign. From what has been seen until now, is that this sign must be positive [1]. H. Minkowski, W. Blaschke initiated this study for the convex hyper surfaces. Their study characterizes, that ovaloids are topological sphere

(Hadamard theorem) where as the closed non-compact surfaces with positive Guass curvature are diffeomorphic to $\mathbb{R}^2$. Further, both are characterized as boundaries of convex domains in Euclidean space [2].

A compact surface with constant mean curvature which is star shaped with respect to given point must be a sphere centered at this point.

## CONCLUSIONS

The above results for ovaloids also poses further questions in case of compact surfaces. Which compact surfaces have constant mean curvature? But the global problems in this regard that is for the mean curvature are more complicated and proved to be quite interesting. A. D. Alexandrov in 1950's and 60's proves an interesting result that the only compact surfaces with constant mean curvature are again spheres. The method that Alexandrov developed was entirely different as compared to the ones given by Liebmann and Jellet. Here, we come across relating geometry with PDE's. Thus the question of determining compact surfaces with constant mean curvatures will lead us to answer one of the oldest questions posed by the Greeks. Namely the isoperimetric problem. This problem may be stated as follows. Which among all the compact surfaces in $\mathbb{R}^3$ whose inner domains have a given volume which has the least area? The answer was sphere and was known to them. H.A. Schwartz showed it based on the ideas of Steiner and Minkowskiss, who had looked for the solution among convex domains [3].

## REFERENCES

1. Ravis Kulkarni, some curvature functions and their arithmetic properties (Dedicated to S.S. Chern)
2. W. Blaschke, Frenets Formeln furden Raum von Riemann, math Zeit, 6 (1921) pp.94-97.
3. Hadwiger, D. Ohmann, Brunn-Minkowskischer Satz and isoperimetric, math. z., 66(1956), pp. 1-8.